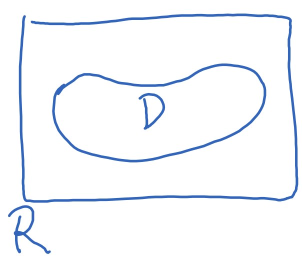
Lecture 15

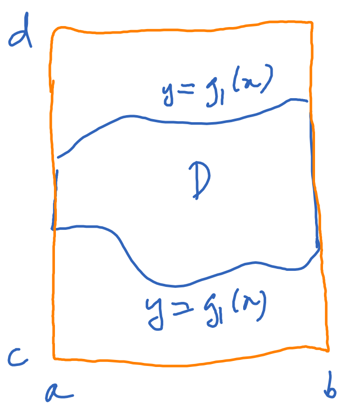
Thursday, March 4, 2021 4:08 PM

* Prayer
* Spiritual thought
* Answering questions ....

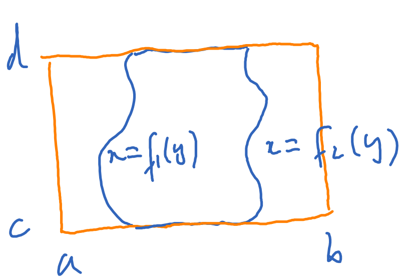
Integral over an arbitrary region :



$$\iint_D f(x,y)\,dA \stackrel{\text{def}}{=} \iint_R \tilde{f}(x,y)\,dA$$

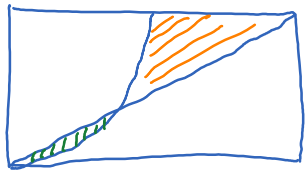


$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$



$$\iint_D f(x,y)\,dA = \int_c^d \int_{f_1(y)}^{f_2(y)} f(x,y)\,dx\,dy$$

Ex

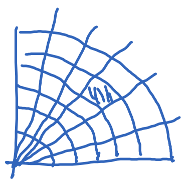


Find the weight of the green and orange plates.

$f(x,y) = x+y \quad (g/cm^2)$

Integral over a polar rectangle

D can be described as $D = \{(r,\theta) : 0 \le r \le 1, \ 0 \le \theta \le \frac{\pi}{4}\}$.



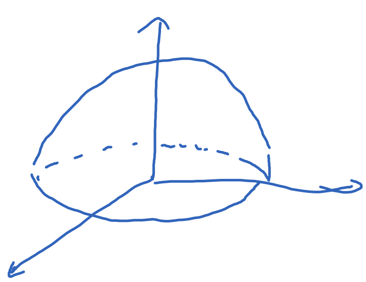
$$\iint_D f(r,\theta)\, dA = \lim \sum_{ij} f(r_i, \theta_j)\, \Delta A_{ij}$$

$$\Delta A_{ij} = \frac{(r_i+\Delta r)^2 \Delta\theta}{2} - \frac{r_i^2 \Delta\theta}{2} \approx r_i \Delta r \Delta\theta$$

$$\iint_D f(r,\theta)\, dA = \lim \sum_{ij} f(r_i, \theta_j)\, r_i \Delta r \Delta\theta$$

$$= \int_c^d \int_a^b f(r,\theta)\, r\, dr\, d\theta .$$

Ex



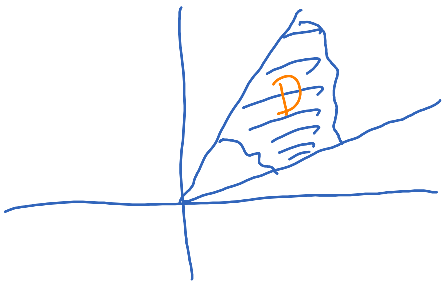
$z = 1 - x^2 - y^2$

Find the volume of the solid.

$$\iint_D (1-x^2-y^2)\, dA .$$

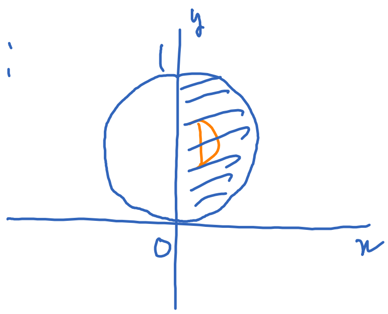
D can be viewed a Type I, Type II region, or a polar rectangle.

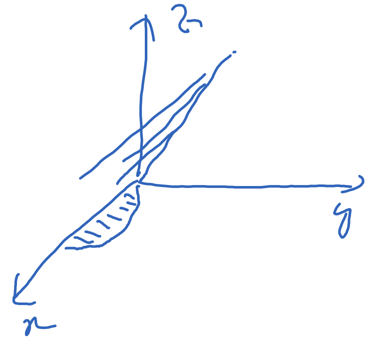
Region determined by $\{(r,\theta): a \le \theta \le b,\ f_1(\theta) \le r \le f_2(\theta)\}$.



$$\iint_D f(r,\theta)\, dA = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} f(r,\theta)\, dr\, d\theta$$

Ex :





$$\iint_D y\, dA = \int_0^1 \int_0^{\sqrt{\frac{1}{2^2} - (y-\frac{1}{2})^2}} y\, dx\, dy \qquad \text{(difficult)}$$

$$D = \{(r,\theta):\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le \sin\theta\}$$

$$\int_0^{\pi/2} \int_0^{\sin\theta} r \sin\theta\, dr\, d\theta = \int_0^{\pi/2} \frac{\sin^3\theta}{2}\, d\theta$$

Use substitution $u = \cos\theta$.

Triple integral

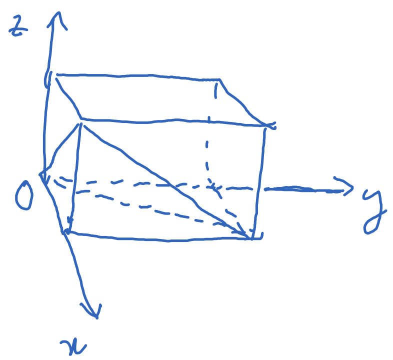
$$\iiint_B f(x,y,z)\,dV = \int_a^b\int_c^d\int_r^s f(x,y,z)\,dz\,dy\,dx$$

$$= \dots \text{ (interchange the order)}$$

Ex: Box $B = [0,1]\times[0,2]\times[0,3]$ has mass density

$$f(x,y,z) = x+y+z$$

what is the mass of the box?



How about the mass of the pyramid section of the box?